Problem set 4

Solve two problems with the written solutions to be handed in by December 4 at 12p.m. (noon).

Those students who have already taken one course on Statistical Physics can swap the solution of one the problems by mentoring one or two students who have not taken any course on Statistical Physics before.

The grade will take into account the written solutions as well as any discussion of these on the week after they were handed in. Successful mentoring will be graded with top marks (18-20).

Problems:

1. Solve the one-dimensional, $-\infty < x < +\infty$, Fokker-Planck equation for a free Brownian particle, i.e., external potential V = 0. Assume that at time t=0 the particle is at x=0. Plot the result as a function of x for several values of t. Calculate the moments $\langle x^n \rangle$ and compare with the results of the Langevin equation approach (<...> denotes the ensemble average).

2. Solve the one-dimensional, $-\infty < x < +\infty$, Fokker-Planck equation for a Brownian particle in the external potential V(x) = mgx, where m is the particle mass, and g the gravitational acceleration. Assume that at time t=0 the particle is at $x = x_0$. Plot the result as a function of x for several values of t, choose the value for x_0.

3. For a free one-dimensional Brownian particle starting at time t = 0 from x = 0, determine the probability density $P_x(t)$ that the particle will reach for the first time a given point x at a time in the interval (t, t + dt). [Remark: $P_x(t)$ dt is the probability that the particle will get from x=0 to the point x, for the first time, in a time in the interval (t, t+dt)].